

Imperical Analysis of TSLA Microstructure Data

FE 570: Market Microstructure

Final Project

Written By:

Amin Labadi

Amod Lamichhane

Naveen Nagarajan

Vineel Pentrala

Chapter 1

Introduction

Financial market microstructure analysis provides a way to examine the underlying dynamics of trading activity, including the forces that drive liquidity, volatility, and informed trading. This project focuses on Tesla, Inc. (TSLA), a stock known for its high trading volume and frequent price movements. These characteristics make it an appropriate subject for understanding market behavior in detail.

Using a dataset of tick-level trades and quotes (TAQ) for Tesla, this study explores three key aspects of market microstructure:

i) The ease of trading, referred to as liquidity.

ii) The stability of prices over time, referred to as volatility.

iii) The likelihood of trades based on material non-public information, referred to as the Probability of Informed Trading (PIN).

To achieve this, measures such as quoted, effective, and realized spreads are computed to study Tesla's intraday liquidity dynamics. Volatility is estimated using intraday trade data, applying methods like realized variance and the Roll model. Additionally, the study derives a PIN metric to assess the level of information asymmetry in Tesla's trading activity.

The insights gained from this analysis are intended to help traders and investors optimize their strategies and make informed decisions. This study is particularly useful for those actively trading Tesla's stock, providing a deeper understanding of its liquidity, price stability, and the role of informed trading in its market activity.

1.1 Goals

The primary goal of this project is to gain a comprehensive understanding of Tesla's stock market microstructure, which is characterized by its high trading volume and price volatility. This is achieved by addressing three key areas of analysis:

1.1.1 Liquidity Analysis

Understanding how efficiently Tesla's stock can be traded is vital for traders and market participants. This involves:

- Calculating spread metrics like quoted spread, effective spread, and realized spread to quantify trading costs.
- Analyzing intraday variations in these metrics to provide insights into Tesla's liquidity dynamics and their implications for trade execution.

1.1.2 Volatility Analysis

Tesla's price stability is a key concern for traders managing risk. This analysis involves:

- Estimating intraday volatility using realized variance and the Roll model to identify the intensity and drivers of price fluctuations.
- Separating fundamental volatility from market noise to better understand Tesla's trading behavior.

1.1.3 Probability of Informed Trading (PIN)

Informed trading influences market efficiency, making it critical to estimate the Probability of Informed Trading in Tesla's stock. This involves:

- Applying likelihood-based models to intraday buy and sell activity segmented into 30-minute intervals.
- Comparing PIN values across different exchanges to uncover variations in informed trading and their implications for liquidity and price discovery.

Chapter 2

Liquidity

2.1 Introduction

Understanding market microstructure is crucial for optimizing trading strategies and improving market efficiency. This chapter explores the liquidity of TSLA by analyzing intraday data over five consecutive days (October 21, 2024 – October 25, 2024). The goal is to identify patterns in liquidity and examine how large trades and significant price movements affect the bid-ask spread, depth, and order imbalance.

2.2 Data and Methodology

2.2.1 Data Collection

We utilize high-frequency intraday data for TSLA, collected over five days, from October 21, 2024, to October 25, 2024. The dataset includes timestamps, bid prices, ask prices, bid sizes, ask sizes, and trade volumes at nanosecond intervals. The data is sourced from the Refinitive Historical Tick DB.

2.2.2 Liquidity Measures

We calculate the following liquidity measures:

- **Midpoint:** The average of the bid and ask prices, representing the best available price in the market.
- **Quoted Spread:** The difference between the bid and ask prices, an indicator of market liquidity.

- **Effective Spread:** Twice the absolute difference between the price at which the trade occurs and the midpoint.
- **Depth:** The sum of the bid and ask sizes, reflecting the market's depth at a given price level.
- **Order Imbalance:** The difference between the bid size and ask size, divided by the total size, providing insight into the market's directionality.

2.2.3 Period Segmentation

The trading day is divided into five distinct periods:

- Pre-Market Trading (4 AM – 9:30 AM)
- Opening Auction (9:30 AM – 10 AM)
- Midday (10 AM – 3 PM)
- Closing Auction (3 PM – 3:30 PM)
- After-Hours Trading (3:30 PM – 8 PM)

2.2.4 Event-Based Analysis

We also analyze the impact of large trades and price jumps on liquidity. A large trade is defined as any trade volume exceeding the 99th percentile, while a price jump is identified as a price change greater than the 99th percentile of absolute price changes.

2.3 Results

2.3.1 Liquidity Patterns by Trading Period

We analyze liquidity across five distinct periods of the day and calculate the average quoted spread, effective spread, depth, and order imbalance for each period and day. The results (Table 2.1) reveal significant differences in liquidity measures across periods:

Table 2.1: Liquidity Patterns

Day	Period	Avg Quoted Spread	Avg Effective Spread	Avg Depth	Avg Order Imbalance
21/10/24	Pre-Market Trading	0.114	0.0822	17.4	0.0803
21/10/24	Opening Auction	0.0934	0.0484	21.1	-0.116
21/10/24	Midday	0.0525	0.0633	13.2	0.0169
21/10/24	Closing Auction	0.04	0.0345	9.96	0.0469
21/10/24	After-Hours Trading	0.0324	0.0243	12	-0.0443
22/10/24	Pre-Market Trading	0.104	0.0596	9.82	-0.00995
22/10/24	Opening Auction	0.0898	0.0425	6.03	-0.116
22/10/24	Midday	0.0543	0.0463	10.9	0.0326
22/10/24	Closing Auction	0.0353	0.0307	15.2	0.036
22/10/24	After-Hours Trading	0.0311	0.0233	14.4	-0.0558
23/10/24	Pre-Market Trading	0.13	0.0668	8.03	0.0662
23/10/24	Opening Auction	0.0812	0.0445	6.76	0.206
23/10/24	Midday	0.0472	0.0404	10.5	0.026
23/10/24	Closing Auction	0.0305	0.0271	41	0.143
23/10/24	After-Hours Trading	0.103	0.127	14.7	-0.0373
24/10/24	Pre-Market Trading	0.132	0.091	8.11	0.123
24/10/24	Opening Auction	0.125	0.0616	8.91	0.167
24/10/24	Midday	0.0669	0.123	32.5	-0.053
24/10/24	Closing Auction	0.0381	0.0336	31.9	-0.109
24/10/24	After-Hours Trading	0.0379	0.0466	54.2	-0.0818
25/10/24	Pre-Market Trading	0.0998	0.0759	8.91	0.0116
25/10/24	Opening Auction	0.0987	0.0569	7.12	0.0644
25/10/24	Midday	0.0702	0.072	26.5	-0.0438
25/10/24	Closing Auction	0.0478	0.0408	9.94	-0.0269
25/10/24	After-Hours Trading	0.0419	0.0336	14.7	-0.0266

2.3.2 Liquidity Patterns by Time Interval

For finer granularity, we analyze liquidity on a 5-minute interval basis between 4 AM and 8 PM. The results (Figure 2.1) show the average quoted spread and depth over time for each day. We observe noticeable spikes in liquidity which likely reflect increased market activity and order flow.

2.3.3 Event-Based Analysis: Large Trades

The analysis of large trades reveals that liquidity conditions change significantly following a large trade. Specifically, the average quoted spread after large trades is 0.056, compared

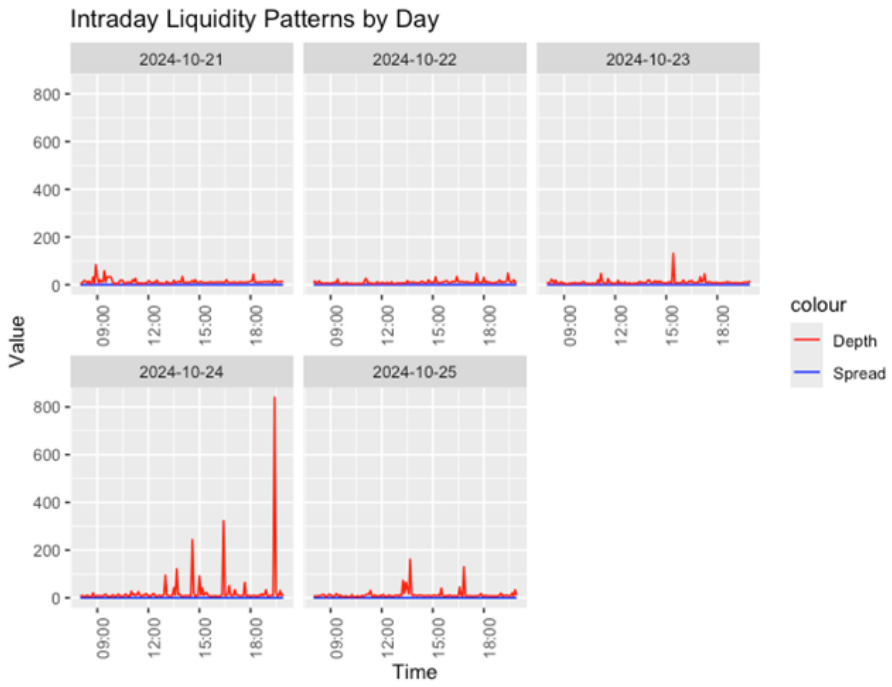


Figure 2.1: Intraday Liquidity Patterns by Day

to a pre-trade average of 0.040. Additionally, average depth after large trades increases to 24.1, suggesting that liquidity is replenished after a large order execution.

2.3.4 Event-Based Analysis: Price Jumps

Price jumps significantly affect liquidity dynamics. The average quoted spread before a price jump is 0.197, while after a price jump, it narrows to 0.056. Depth also decreases before a price jump (14.02) and increases afterward (24.07), suggesting that the market becomes more liquid in response to significant price changes.

Reasons for Price Jumps: The observed price jumps during the study period (October 21–25, 2024) can be attributed to key Tesla-specific and broader market events:

- October 21, 2024: Market reactions to the broader market events during the weekend.
- October 23, 2024: Tesla released its Q3 2024 financial results after the closing bell, which exceeded market expectations with revenue of $25.6 billion and adjusted earnings of $2.11 per share. Despite this, concerns over increased production costs in facilities in Texas and Berlin led to mixed market reactions, causing significant volatility.
- October 24, 2024: Market reactions to the prior day's earnings.

- October 25, 2024: Participants likely hedging their risk for the coming weekend, or reactions to other statements made by Tesla.

2.4 Discussion

The results underline the dynamic nature of intraday liquidity and its sensitivity to market events. The substantial liquidity changes observed during price jumps and large trades suggest an active role of institutional liquidity providers.

High-frequency trading (HFT) firms and market-making algorithms likely contributed to the rapid replenishment of liquidity observed post-event. Their presence ensures:

- Efficient Price Discovery: Tightening spreads after significant price movements.
- Enhanced Market Depth: Increasing available liquidity at various price levels to accommodate elevated trading volumes.

Furthermore, Tesla's unique market position—characterized by high volatility and a mix of retail and institutional investors—amplifies these dynamics.

Retail investor behavior, often driven by news and sentiment, introduces a layer of unpredictability, which institutional algorithms work to mitigate.

Chapter 3

Volatility

3.1 Intraday volatility

Volatility is the tendency for prices to change unexpectedly. Prices change in response to new information about values and in response to the demands of impatient traders for liquidity. Fundamental volatility and Trading volatility are two types of volatility at the microscale.

Seventeen different exchanges are represented in the Reuters tick data, among which the top four exchanges by trades—ADF (Alternative Display Facility), NAX (NASDAQ), DEX (Designated Exchange), and PSE (Philippine Stock Exchange)—are selected for the volatility study.

For the purpose of the study market data is extracted from Reuters tick db from 21st Oct - 25th Oct ,

Table 3.1: Days

DAY 1	21-Oct-2024
DAY 2	22-Oct-2024
DAY 3	23-Oct-2024
DAY 4	24-Oct-2024
DAY 5	25-Oct-2024

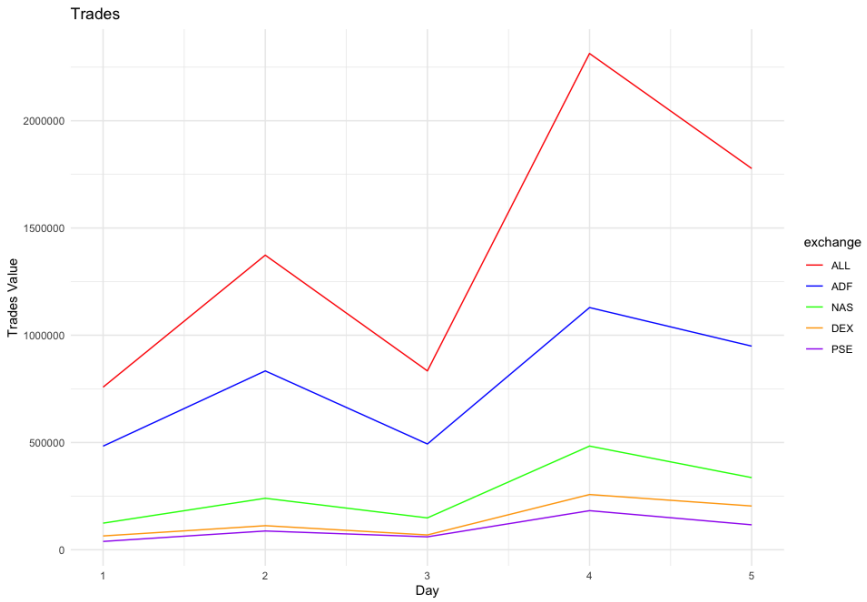


Figure 3.1: Volume across exchanges

3.1.1 Roll model

The trade price $p_t = m_t + \epsilon_t$ has two components:

i) the efficient price m_t and

ii) trading noise contribution ϵ_t.

Roll model volatility of the increments of the efficient price $\sigma_u^2 = \gamma_0 + 2\gamma_1$

$\sigma_{day} = \sqrt{n_{trades} * \sigma_u}$

In the formula $\sigma_u^2 = \gamma_0 + 2\gamma_1$, the second term $2\gamma_1$ is negative and subtracts the trading activity contribution from the total volatility.

We obtained the fundamental volatility of the stock price $\sigma_{ann} = \frac{\sigma_u}{\bar{S}} \times \sqrt{n_{trades}} \times \sqrt{252}$

The total volatility is obtained by replacing σ_u^2 with $\gamma_0 = var(\Delta p_t)$, the total price volatility. This gives

$\sigma_{ann}^{total} = \frac{\sqrt{\gamma}_u}{\bar{S}} \times \sqrt{n_{trades}} \times \sqrt{252}$

Autocorrelation at lag 1 suggests a short-term dependency between consecutive differenced price values. For lags beyond 1, the autocorrelation values are within the confidence bounds, implying no statistically significant autocorrelation. The pattern in consistent across all five days.

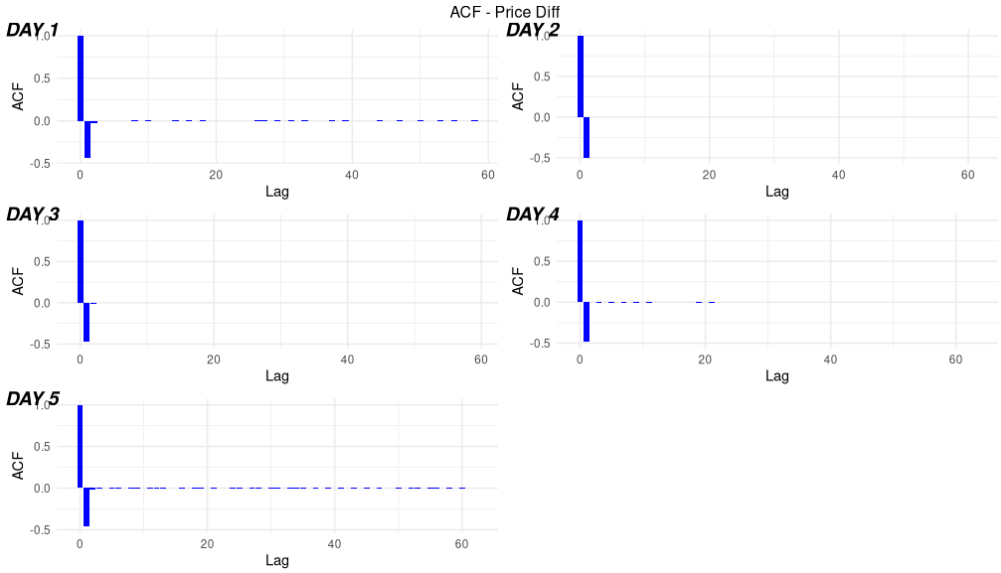


Figure 3.2: Autocorrelation function (ACF) of price differences.

The ACF of trade signs shows a slow, gradual decay over the lags, suggesting a persistent autocorrelation in the trade signs.

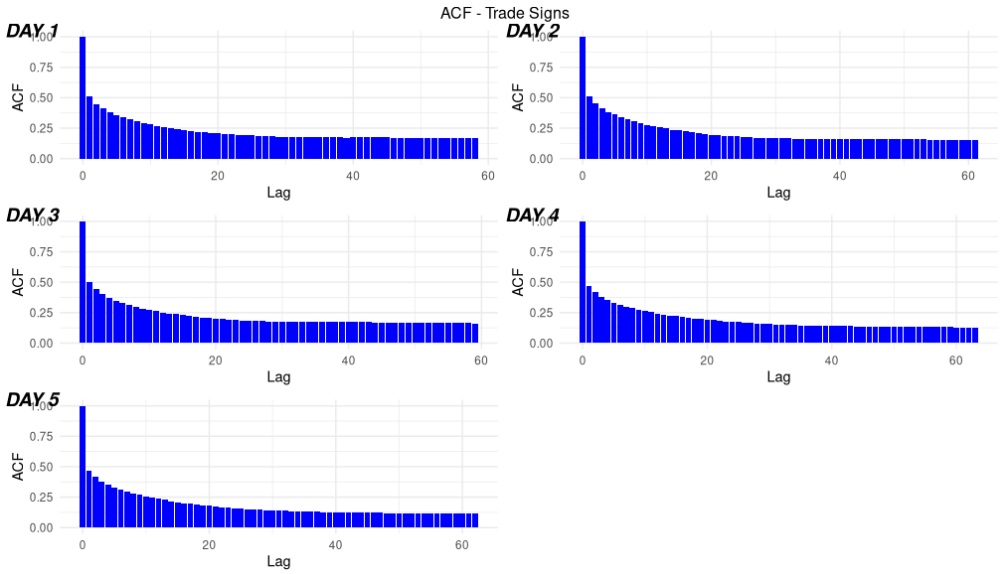


Figure 3.3: Autocorrelation function (ACF) of trade signs.

For most days, the mid prices closely track the trade prices, indicating a relatively efficient market with minimal divergence between the actual trades and the theoretical mid prices. Sudden spikes in the trade price (red line) suggest high volatility during specific periods.

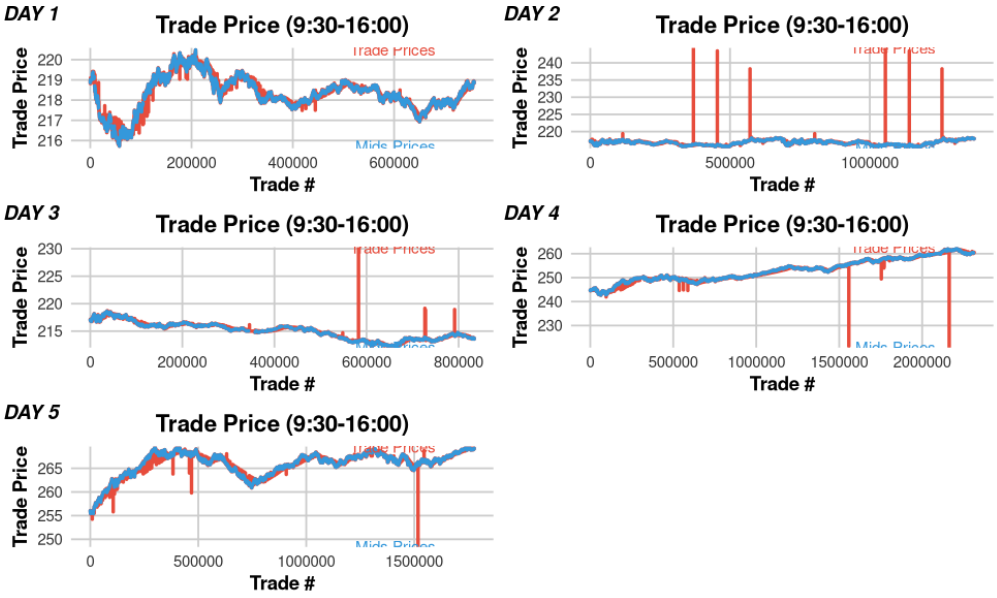


Figure 3.4: Price plot over all days.

Across all days, most trades have relatively low volumes (a consistent baseline). However, sporadic spikes in trade size suggest periods of high trading activity or large block trades. The volume baseline remains relatively constant across the observed days, with no major shifts in trading patterns except for the spikes. Spikes may indicate either large institutional trades or block trades, reaction to news or market events during specific times of the day.

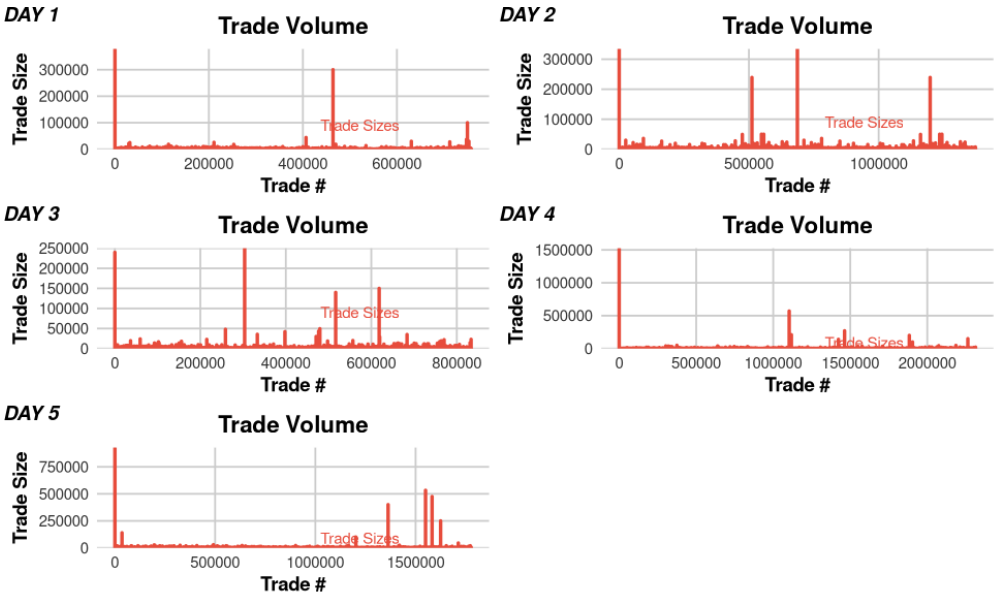


Figure 3.5: Volume plot over all days.

The red line (representing "ALL") generally shows the highest roll values across all days, indicating that the combined market volume or activity across all exchanges is consistently the highest.

There is a dip in roll values for Day 3 across all exchanges, suggesting reduced market activity or lower trading volumes.

By Day 5, all exchanges (ADF, NAS, DEX, PSE) converge to similar roll values, reflecting consistent trading behavior.

A notable drop on Day 3 might indicate a day of lower market participation, possibly due to external events or lack of significant trading catalysts.

On Day 5, roll values across exchanges align, suggesting synchronized trading activity across markets.The convergence of roll values across exchanges by Day 5 indicates increased market synchronization, likely due to alignment in trading strategies or external factors driving uniform behavior.

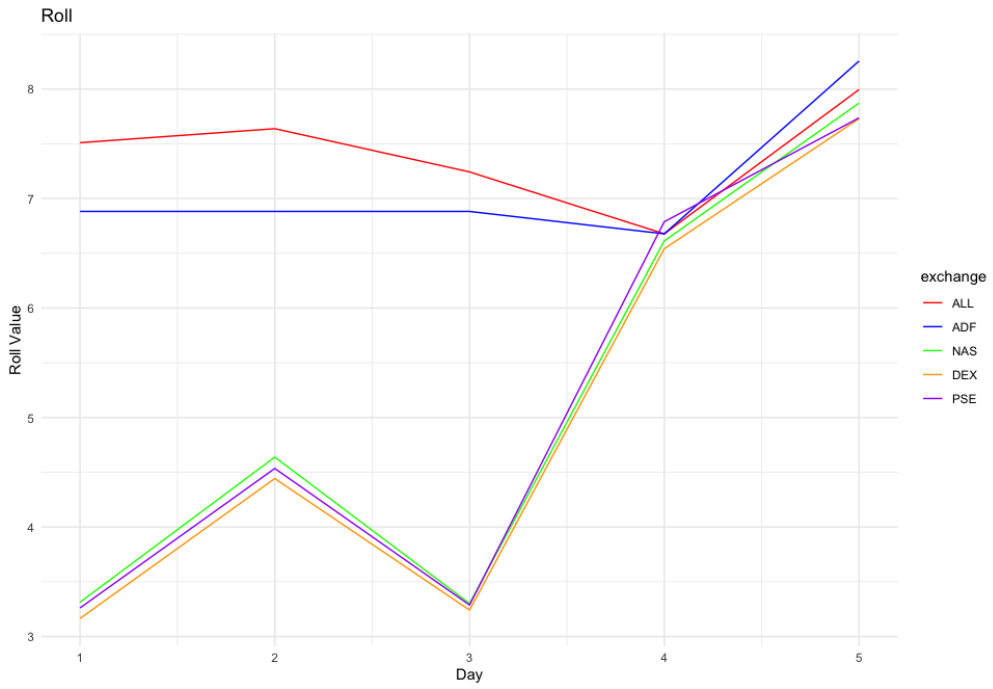


Figure 3.6: σ_{day}^{roll} across exchanges

Roll model estimate - across different exchanges

Roll model has two parameters c and σ_u^2. These are estimated from variance and first-order auto-covariances of price changes Δp_t. The first order auto-covariance is $\gamma_1 = -c^2$, the spread estimate doesn't take into account trade directions and an improved c is estimated by taking regression of prices against mid price and trade direction.

$$prices \sim mid.price + c_{improved} * tradeSigns$$

Based on all trades for $\gamma_1 = -0.000646$ Day-3 shows c=0.025421, which shows spread estimate is $2 \times c$=0.050842 cents. Improved roll model gives the spread estimate as $c_improved$=0.010721 cents. Roll model spread estimate is typically used when bid ask spread for a exchange is not available.

```
td <- getTradeDirection(tqdata)
pr <- as.numeric(tqdata$PRICE)
dpr <- diff(pr)
deps <- diff(td)
mids <- (as.numeric(tqdata$OFR) + as.numeric(tqdata$BID))/2
dm <- diff(mids)
```

```
(fit.lm <- lm(dpr ~ dm + deps))
fit.lm$coeff[3]
```

Table 3.2: Day 1

Parameter	**ALL**	**ADF**	**NAS**	**DEX**	**PSE**
γ_0	0.000553	0.000804	0.000094	0.000174	0.000278
γ_1	-0.000239	-0.000353	-0.000003	-0.000009	-0.000002
σ_u^2	0.000074	0.000098	0.000088	0.000156	0.000274
σ_{ann}	119.219577	109.254274	52.592650	50.237125	51.767872
σ_{ann}^{total}	325.135041	312.769808	54.247270	53.177290	52.134473
c_{roll}	0.015475	0.018782	0.001681	0.003063	0.001395
$c_{improved}$	0.012005	0.013175	0.007682	0.009389	0.008555

Table 3.3: Day 2

Parameter	**ALL**	**ADF**	**NAS**	**DEX**	**PSE**
γ_0	0.006223	0.010163	0.000101	0.000196	0.000241
γ_1	-0.003090	-0.005056	-0.000006	-0.000010	-0.000003
σ_u^2	0.000042	0.000051	0.000090	0.000176	0.000235
σ_{ann}	121.249242	103.788033	73.631340	70.529922	71.988734
σ_{ann}^{total}	1467.618732	1461.371830	78.054486	74.356083	72.965617
c_{roll}	0.055592	0.071105	0.002355	0.003131	0.001791
$c_{improved}$	0.012128	0.013327	0.007795	0.009612	0.008996

Table 3.4: Day 3

Parameter	**ALL**	**ADF**	**NAS**	**DEX**	**PSE**
γ_0	0.001355	0.002234	0.000080	0.000164	0.000177
γ_1	-0.000646	-0.001071	-0.000003	-0.000006	-0.000001
σ_u^2	0.000063	0.000092	0.000073	0.000152	0.000180
σ_{ann}	114.984459	107.055343	52.409247	51.459669	52.188730
σ_{ann}^{total}	533.660641	526.873995	54.597910	53.418574	51.822663
c_{roll}	0.025421	0.032723	0.001769	0.002429	0.001120
$c_{improved}$	0.010721	0.011722	0.007268	0.008897	0.007830

Table 3.5: Day 4

Parameter	**ALL**	**ADF**	**NAS**	**DEX**	**PSE**
γ_0	0.003904	0.007744	0.000143	0.000236	0.000295
γ_1	-0.001899	-0.003771	-0.000012	-0.000026	-0.000012
σ_u^2	0.000106	0.000202	0.000118	0.000185	0.000271
σ_{ann}	248.546880	239.987100	120.140460	109.610159	111.551916
σ_{ann}^{total}	1508.654637	1484.461833	131.880548	123.852757	116.345054
c_{roll}	0.043579	0.061407	0.003485	0.005063	0.003447
$c_{improved}$	0.015987	0.017723	0.010703	0.012761	0.012863

Table 3.6: Day 5

Parameter	**ALL**	**ADF**	**NAS**	**DEX**	**PSE**
γ_0	0.002313	0.004137	0.000187	0.000293	0.000449
γ_1	-0.001069	-0.001939	-0.000011	-0.000026	-0.000004
σ_u^2	0.000174	0.000259	0.000166	0.000240	0.000441
σ_{ann}	279.512621	249.103036	118.465563	111.111515	113.708563
σ_{ann}^{total}	1017.850330	994.706778	125.962374	122.613997	114.663811
c_{roll}	0.032699	0.044029	0.003288	0.005114	0.001929
$c_{improved}$	0.018419	0.020353	0.012339	0.014418	0.014067

3.1.2 Realized Variance

This method uses trade prices sampled at lag q. The daily volatility estimate at lag q is given by

$$\sigma^2_{Day}(q)(q\Delta) = Var(\Delta p_q), \Delta p_q := p_{t+q} - p_t$$

Typically a lag of 5 minutes is sufficient for the trading noise term ϵ_t to average out. The required lag can be estimated visually from the signature plot, which is a graphical representation of $\sigma Day(q)$ vs q. This plot should have a plateau where the daily volatility is independent of lag.

The method is implemented in the R code by the function realizedVar(q) which computes the realized variance rv(q) of the trade price pt for the entire trading day at lag q in trading time

$$realizedVar(q) = rv(q) = \frac{1}{q}\sum_{i=1}^{\lfloor n/q \rfloor} (p_{t+q} - p_t)^2$$

From this we determine the daily volatility at lag q as $\sigma Day(q) =$ rv(q).

Signature plots is stable across all 5 days. Considering all exchanges $q_{5min} = 63.29$ is seen to be highest on day 5. Figure 8 shows us that by day 5 the q_{5min} hit its peak. $q_{5min} = 68.16$ on ADF is higher compared to ALL exchanges combined, possible reason could be due to better pricing and spread available on ADF.

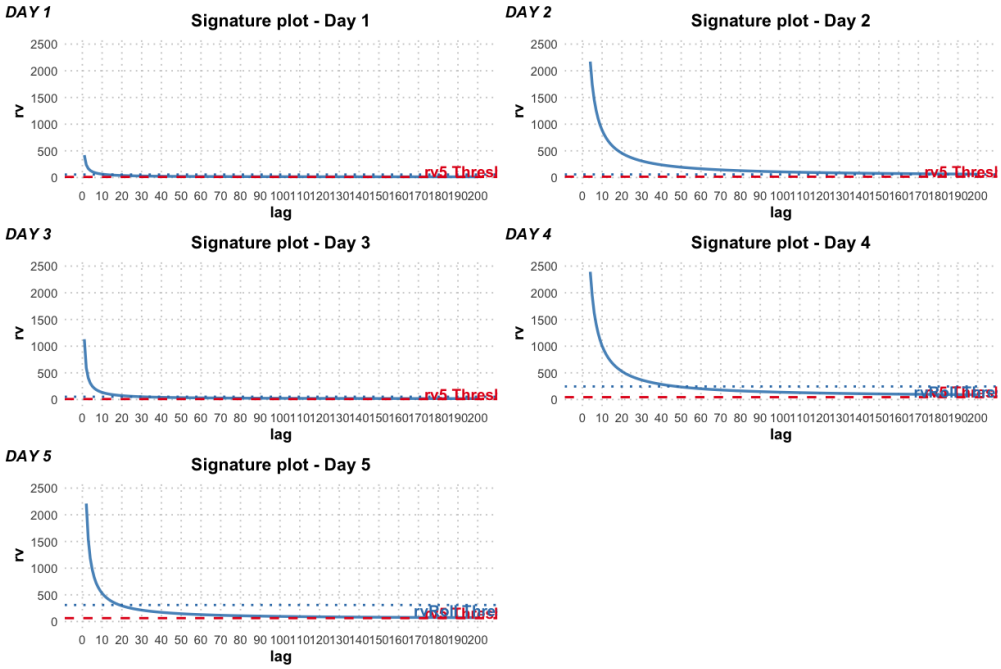


Figure 3.7: Signature plot

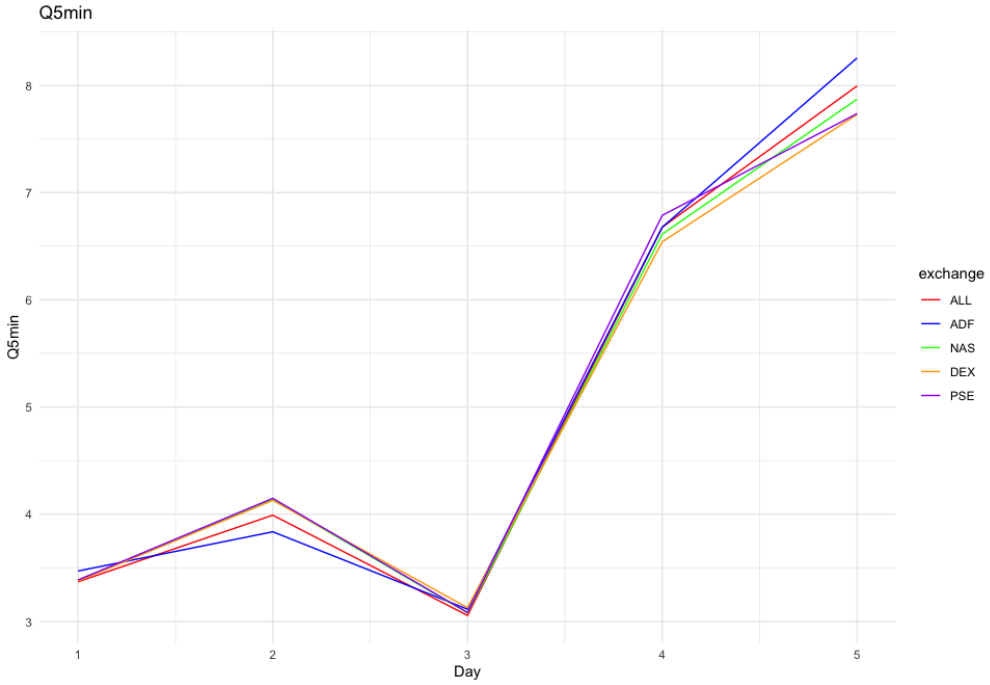


Figure 3.8: q_{5min} *plot across exchanges*

Roll Model vs Realized Variance

Table 3.7: Day 1

Type	**ALL**		**ADF**		**NAS**		**DEX**		**PSE**	
	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ
q_1	419.49	20.48	388.19	19.70	11.33	3.37	11.22	3.35	10.79	3.28
q_2	237.95	15.43	217.78	14.76	11.33	3.37	10.62	3.26	10.70	3.27
q_{5min}	11.35	3.37	12.04	3.47	11.43	3.38	11.40	3.38	11.47	3.39
	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}
Roll model	56.40	7.51	47.37	6.88	10.98	3.31	10.01	3.16	10.63	3.26
Trades	758112		483079		124082		64315		38855	

Table 3.8: Day 2

Type	**ALL**		**ADF**		**NAS**		**DEX**		**PSE**	
	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ
q_1	8547.23	92.45	8474.62	92.06	24.18	4.92	21.94	4.68	21.13	4.60
q_2	4302.79	65.60	4258.68	65.26	22.83	4.78	20.84	4.56	20.85	4.57
q_{5min}	15.92	3.99	14.71	3.84	17.12	4.14	17.05	4.13	17.21	4.15
	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}
Roll model	58.34	7.64	42.75	6.88	21.51	4.64	19.74	4.44	20.56	4.53
Trades	1373398		833862		240038		112176		87602	

Table 3.9: Day 3

Type	**ALL**		**ADF**		**NAS**		**DEX**		**PSE**	
	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ
q_1	1130.13	33.62	1101.57	33.19	11.83	3.44	11.32	3.37	10.66	3.26
q_2	591.30	24.32	573.52	23.95	11.36	3.37	10.92	3.30	10.73	3.28
q_{5min}	9.33	3.05	9.69	3.11	9.48	3.08	9.78	3.13	9.50	3.08
	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}	rv_{Roll}	σ_{day}^{roll}
Roll model	52.47	7.24	45.48	6.88	10.90	3.30	10.51	3.24	10.81	3.29
Trades	833824		493149		148489		69071		60175	

Table 3.10: Day 4

Type	**ALL**		**ADF**		**NAS**		**DEX**		**PSE**	
	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ
q_1	9031.90	95.04	8744.54	93.51	69.02	8.31	60.87	7.80	53.72	7.33
q_2	4638.52	68.11	4486.54	66.98	63.15	7.95	54.28	7.37	51.55	7.18
q_{5min}	44.59	6.68	44.59	6.68	43.73	6.61	42.79	6.54	46.08	6.79
	rv_{Roll}	σ^{roll}_{day}	rv_{Roll}	σ^{roll}_{day}	rv_{Roll}	σ^{roll}_{day}	rv_{Roll}	σ^{roll}_{day}	rv_{Roll}	σ^{roll}_{day}
Roll model	245.14	15.66	228.55	6.88	57.28	7.57	47.68	6.90	49.38	7.03
Trades	2313397		1129191		483448		257414		182361	

Table 3.11: Day 5

Type	**ALL**		**ADF**		**NAS**		**DEX**		**PSE**	
	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ	rv(q)	σ
q_1	4111.19	64.12	3926.35	62.66	62.96	7.93	59.66	7.72	52.17	7.22
q_2	2210.60	47.02	3926.35	62.66	59.32	7.70	54.33	7.37	51.74	7.19
q_{5min}	63.92	8.00	68.16	8.26	61.97	7.87	59.75	7.73	59.86	7.74
	rv_{Roll}	σ^{roll}_{day}	rv_{Roll}	σ^{roll}_{day}	rv_{Roll}	σ^{roll}_{day}	rv_{Roll}	σ^{roll}_{day}	rv_{Roll}	σ^{roll}_{day}
Roll model	310.03	17.61	246.24	15.69	55.69	7.46	48.99	7.00	51.31	7.16
Trades	1777544		949185		336278		203923		116282	

3.2 Results

The results from both the Roll model and realized variance measures indicate that ADF absorbs the impact of volatility much faster than other venues. However, in contrast, the spread estimate for ADF is notably higher compared to other exchanges, which could be attributed to market imbalances and the execution of large trades.

Chapter 4

Probability of Informed Trading (PIN)

This report presents an analysis of trading data for TSLA from October 21 - 25, 2024, focusing on intraday trade patterns, and the Probability of Informed Trading (PIN). The trading day (9:30 AM to 4:00 PM) was divided into thirteen 30-minute intervals. Each interval has a number of buys and number of sells.

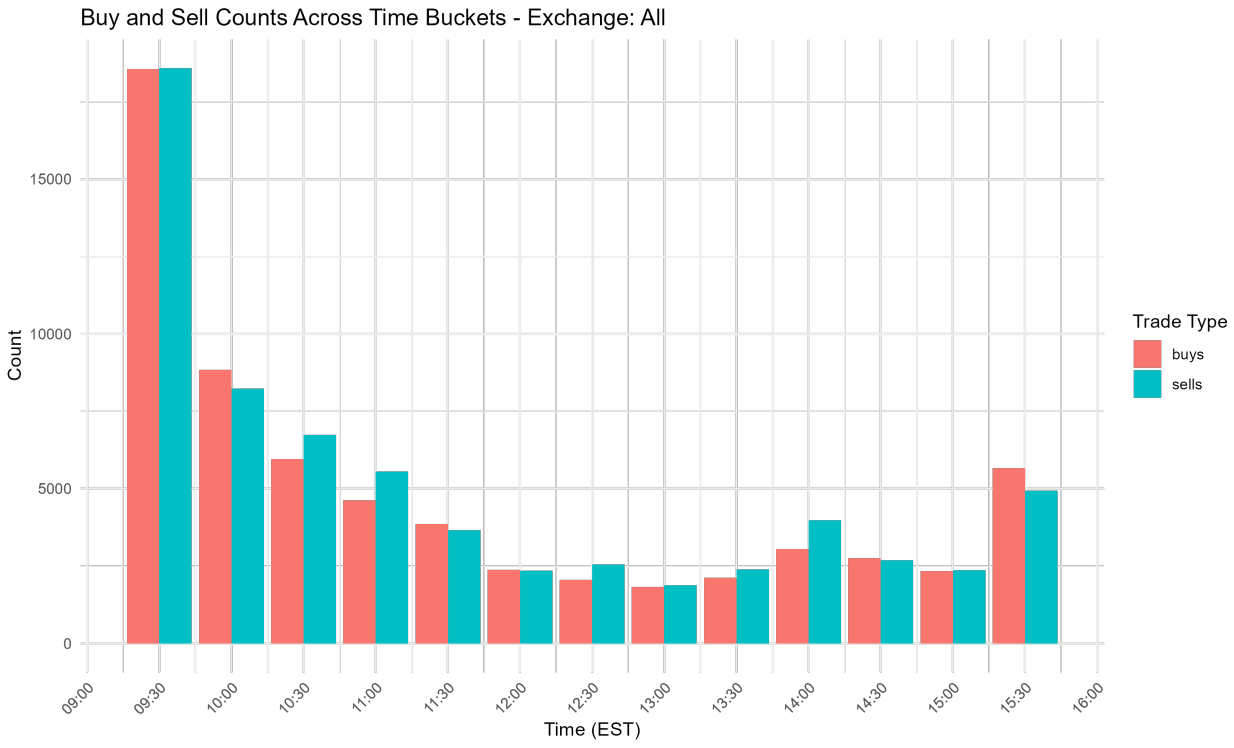


Figure 4.1: Trade Distribution Across Time Buckets (Day 1)

Using these time buckets, we are able to create a data variable. The data variable can

then be used to calculate α, δ, μ, ϵ_b, ϵ_s, and the PIN using models from previous studies available through the 'Infotrad' package in R. EA, YZ, and GAN models are used to calculate the PIN with the likelihood of LK and EHO.

Model	α	δ	μ	ϵ_b	ϵ_s	PIN
YZ_EHO	0.9894	0.9363	644.5832	6885.875	7475.252	0.0781
YZ_LK	0.1538	0.0000	10365.1865	3332.819	5072.691	0.1043
GAN_EHO	0.5906	0.6760	563.4562	4119.178	5731.847	0.0327
GAN_LK	0.2308	1.0000	7945.6338	4927.453	3239.090	0.1834
EA_EHO	0.8712	0.7260	335.6257	7078.757	7360.908	0.0198
EA_LK	0.1538	1.0000	9852.5836	4927.461	3556.910	0.1516

Table 4.1: Estimated Model Parameters and resulting PIN values for different configurations. (Day 1)

4.1 InfoTrad

The optimal parameter estimates in our study were informed by the findings of a 2018 paper, which highlighted the performance of different models in estimating PIN components. Since the true values of the model parameters (θ) were not available for direct evaluation, we relied on insights from the survey conducted by [1].

The study evaluated various methods using Mean Absolute Error (MAE) as a proxy for accuracy. It found that the `GAN()` method with the factorization "EHO" was most effective in capturing the true values of α (probability of information events), δ (probability of informed traders participating), ϵ_b, and ϵ_s. Meanwhile, the `YZ()` method with `EHO` factorization performed best in estimating μ (arrival rate of informed trades). Finally, `YZ()` with `LK` provided the most reliable PIN estimates. The following table shows the estimates of the parameters and PIN value chosen.

PIN Across Different Days

The analysis of TSLA data over the course of the week revealed that, on average, there is an unconditional probability of approximately 8.69% that a randomly selected trader on any given day is informed. Notably, following the earnings announcement on Day 3 after hours, the Probability of Informed Trading (PIN) decreased. This decline can be attributed to a reduction in information asymmetry between informed and uninformed traders, as the earn-

Table 4.2: PIN Values Across Different Days of the Week (Using `InfoTrad`)

Day	PIN	α	δ	μ	ϵ_b	ϵ_s
1	0.1043	0.5906	0.6760	644.58	4119.18	5731.85
2	0.1087	0.4569	0.1320	462.13	10895.46	11182.22
3	0.1097	0.7295	0.6760	735.97	8876.13	9227.62
4	0.0567	0.4727	0.6881	284.13	22357.98	25791.23
5	0.0553	0.2976	0.5989	246.26	14454.95	12024.62

ings announcement made previously private or insider information public. Additionally, the increase in trading volume likely diluted the impact of informed trading, further contributing to the drop in PIN.

PIN Across Different Exchanges

Probability of Informed Trading (PIN) can vary across different exchanges for the same asset. For such exploration, PIN is calculated using the InfoTrad methodology applied to 13 time buckets, as detailed above. Figure 4.2 shows the variation of PIN values across different exchanges for each day.

Among the exchanges analyzed, certain exchanges exhibit higher PIN values even after earnings announcements. This disparity in PIN values across exchanges offers valuable insights into how different venues may attract or facilitate trading by informed participants. Exchanges with a higher concentration of informed traders could indicate the presence of more sophisticated trading strategies, such as those employed by institutional investors, high-frequency traders, or individuals utilizing insider information. Conversely, exchanges with lower PIN values may be associated with more retail-driven or uninformed trading environments.

Understanding these differences can help market participants and regulators identify which exchanges are more likely to experience informed trading, allowing for better monitoring and potentially more effective regulation. Additionally, this knowledge can assist traders in selecting the most appropriate venues for their trading strategies based on the level of informed trading activity.

4.2 PINstimation

A study by Ersan and Ghachem [2] introduced a refined methodology for estimating the adjusted Probability of Informed Trading (AdjPIN). This approach leverages the `adjpin()` function in the `PINstimation` package, which applies a Maximum Likelihood Estimation

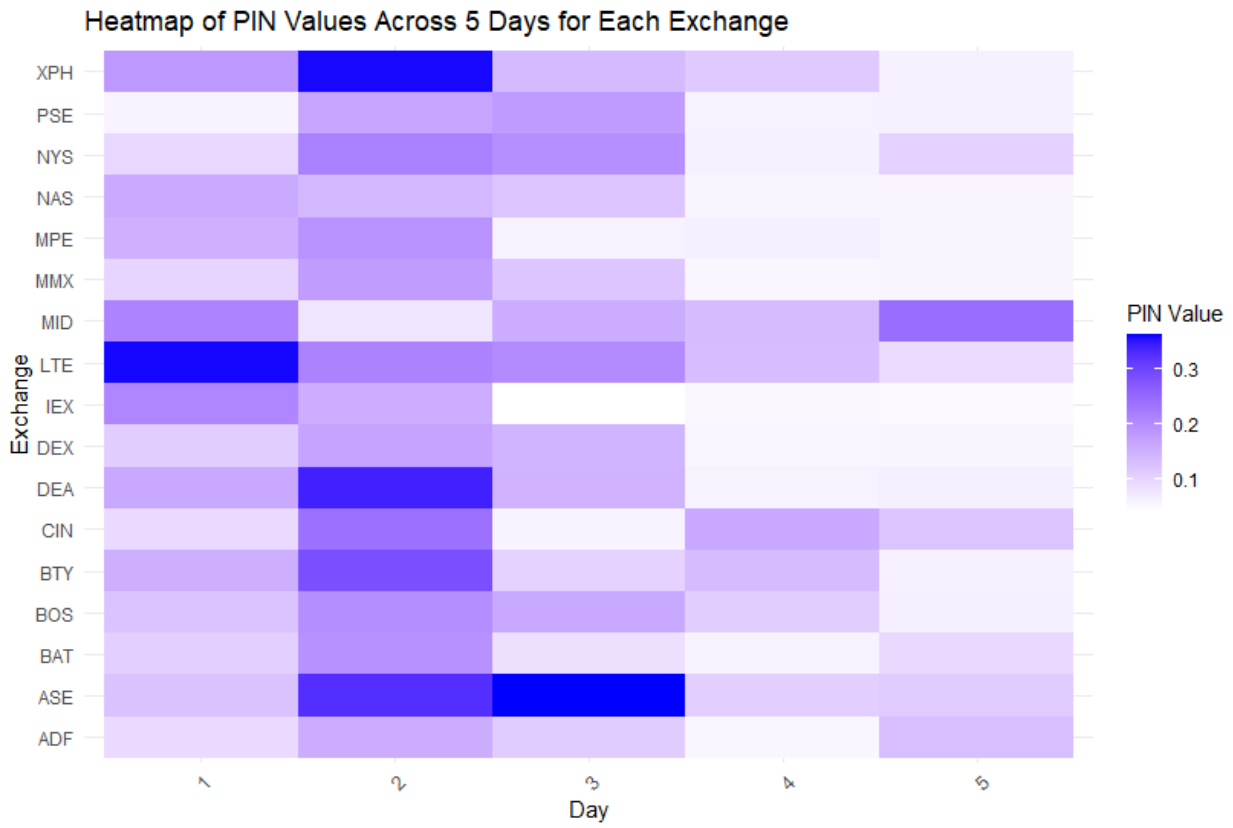


Figure 4.2: Heatmap of Probability of Informed Trading (PIN) across Different Exchanges

(MLE) algorithm. By evaluating 64 combinations of θ parameters, this method enhances the accuracy and robustness of PIN calculations.

The `PINstimation` methodology offers a systematic approach to adjusting PIN values, addressing potential biases and limitations in traditional models. Table 4.3 compares the PIN values calculated using this approach with those derived from the `InfoTrad` package.

Table 4.3: Comparison of PIN Values (Using `InfoTrad` and `PINstimation`)

Day	`InfoTrad` PIN	`PINstimation` PIN
1	0.1043	0.1482
2	0.1087	0.0838
3	0.1097	0.0762
4	0.0567	0.0661
5	0.0553	0.1383

The analysis indicates that the `PINstimation` package produces slightly adjusted PIN values compared to the `InfoTrad` methodology, with an average adjusted PIN of approximately 0.1025. These findings demonstrate close alignment with estimates from `InfoTrad`,

further validating the robustness of the results.

Notably, the `PINstimation` package reveals higher PIN values on Mondays and Fridays. This trend could be attributed to informed traders acting on information accumulated over the weekend, driving higher PIN values at the beginning of the week. Similarly, the elevated PIN on Fridays may reflect traders hedging against the uncertainty of potential news events over the weekend, prompting increased trading activity.

Conclusion

This project provides a detailed examination of Tesla's stock microstructure through the lenses of liquidity, volatility, and the probability of informed trading. By analyzing tick-level trade and quote data, we uncovered valuable insights into the trading dynamics and market behavior of one of the most actively traded stocks in the financial markets.

The liquidity analysis highlighted Tesla's ability to sustain high trading activity with varying levels of efficiency throughout the day. Narrow spreads during midday hours reflect heightened liquidity, while wider spreads at the market open and close underscore the impact of volatility and reduced market depth. These patterns emphasize the importance of timing and strategy in optimizing trade execution costs for traders and investors.

Volatility estimation offered deeper insights into Tesla's price stability. Using realized variance, signature plots, and the Roll model, we identified key contributors to price fluctuations, such as market noise and fundamental drivers. Our findings reveal predictable intraday volatility patterns, which are critical for managing trading risks and developing effective strategies in a dynamic market environment.

The PIN analysis provided insights into informed trading within Tesla's market activity. By applying likelihood-based models and evaluating adjusted PIN values across 64 parameter combinations, we achieved a deeper understanding of trading behavior. Exchange-level analysis revealed significant variations, with LTE exhibiting higher PIN values indicative of institutional trading and ADF showing lower values driven by retail activity. Additionally, day-specific trends emerged, such as lower PIN values following Tesla's earnings announcements due to reduced information asymmetry, and higher PIN values on Mondays and Fridays, reflecting informational lags or pre-news hedging. These findings highlight the nuanced role of informed trading in shaping Tesla's market behavior and its implications for market efficiency.

Overall, this project demonstrates the critical value of analyzing market microstructure data in uncovering the hidden mechanics of trading. By systematically evaluating liquidity, volatility, and informed trading, we not only deepened our understanding of Tesla's trading behavior but also provided actionable insights for traders aiming to optimize strategies,

investors assessing risks, and policymakers ensuring market fairness. Furthermore, the robust PIN analysis enhances this study's applicability, offering a comprehensive framework for understanding informed trading in dynamic financial markets.

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