American Options on SMCI

FE-620

December 17, 2024

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1 Problem setting and choice of underlying stock

In this paper, we study the pricing of American options on SMIC from 02-Oct-2023 to 01-Oct-2024. We do an analysis of the SMIC stock price over a year, and see the effects of options pricing over a volatile market. Super Micro Computer Inc (SMCI) is an IT company based on San Jose, California, with a market in high-performance servers, including the Cloud, data centers and 5G internet. One of SMCI's most recent developments in the market is in artificial intelligence, being apart of the list of stocks in the 2024 AI bubble. As seen in Fig 1., we will study SMCI's volatile change in the market, and it effects on trading American options.

The payoff schema for an American option is as follows:

$$\text{Call Price: } C = max(S_T - K, 0)$$

$$\text{Put Price: } P = max(K - S_T, 0)$$

SMCI does not offer Dividends $D = 0$, so the American options pricing is restricted by put-call parity:

$$\text{American Put-Call Parity } (D = 0)\text{: } S_0 - K < C - P < S_0 - Ke^{-rT}$$

The model we use for this paper is the Black-Scholes model using binomial tree. The formula for estimating stock prices in this model can be formulated as:

$$dS(t) = rS(t)dt + \sigma S(t)dW(t)$$

The binomial tree option pricing estimation requires three parameters, which we can estimate from relevant stock data:

- Spot stock price S_0 of SMCI on 02-Oct-2023 closing date.
- Volatility σ, estimated from log returns of stock prices.
- Risk-free rate r, based on the 13-week T-bill rate.

Furthermore, we will use the binomial tree method to estimate Greeks to further analyze the volatility in SMCI.

2 Market Data Analysis

We download market data from YahooFinance, this includes SMCI closing market data and 13-week T-bill rate from ^IRX ticker from 02-Oct-2023 to 01-Oct-2024.

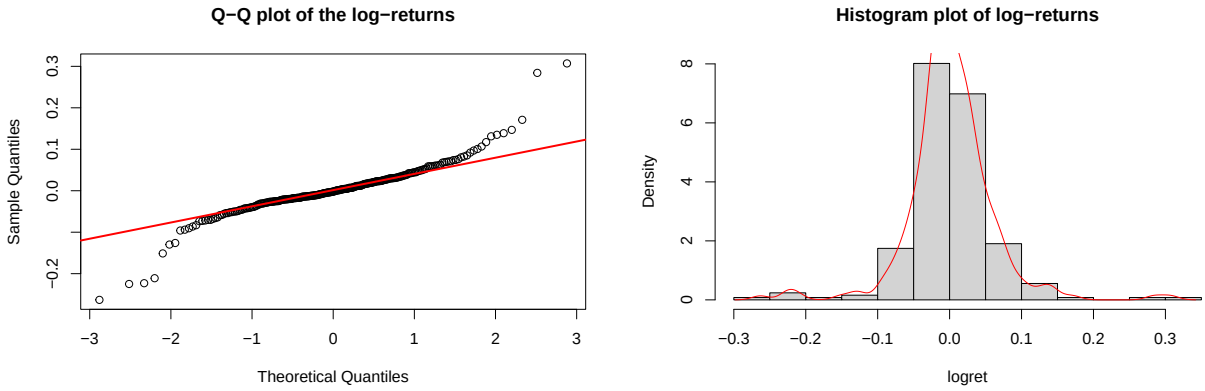
2.1 Time series analysis of stock price S_i

The plot in Fig 1. shows the closing prices for SMCI across $T = 1Y$, 02-Oct-2023 to 01-Oct-2024. We calculate daily log returns via $u_i = log(S_i/S_{i-1})$. The Black Scholes formula we assume log-returns are idd normally distributed random variables with mean $\mu = r - \frac{1}{2}\sigma^2$ and variance $Var(u) = \sigma^2\tau$, and time step τ is based on total business days $d = 252$, $\tau = \frac{1}{252}$.

As observed in Fig 2., the a line passes through the QQ-plot which agrees with the closing data being normally distributed. Furthermore, Fig 2. shows a significant skew in the data, with most of the values in the mean $\mu = 0$ of the distribution. These deviated tails agree with taking the log-normal of market data.



Figure 1: SMCI closing prices



(a) QQPlot (b) Histogram

Figure 2: Log-returns, data has center-skew

2.2 Estimating Parameters from Times Series

Estimate historical volatility

We estimate historical volatility parameter across τ time steps, this is formulated as:

$$\hat{\sigma} = \sqrt{\frac{1}{\tau} Var(u_i)} = \sqrt{252} * \text{stdev}(u_i) = 0.9610093$$

and the historical volatility error:

$$\frac{\delta\hat{\sigma}}{\hat{\sigma}} = \frac{1}{\sqrt{2n_{days}}} = 0.0428068$$

In this case, $n_{days} = 252$ is the total days in our market close period. As evident from Fig 1. volatility increases when we include the volatile range of the market data, from the 6M+ period range. Furthermore, Fig 3. shows the increasing historical volatility error with a larger range due to the volatility in market close data. However, to capture this volatile data and study its effects on options pricing, we will use the full range $n_{days} = 252$. This gives us a historical volatility estimate of: $(0.9610093 \pm 0.0428068)\%$

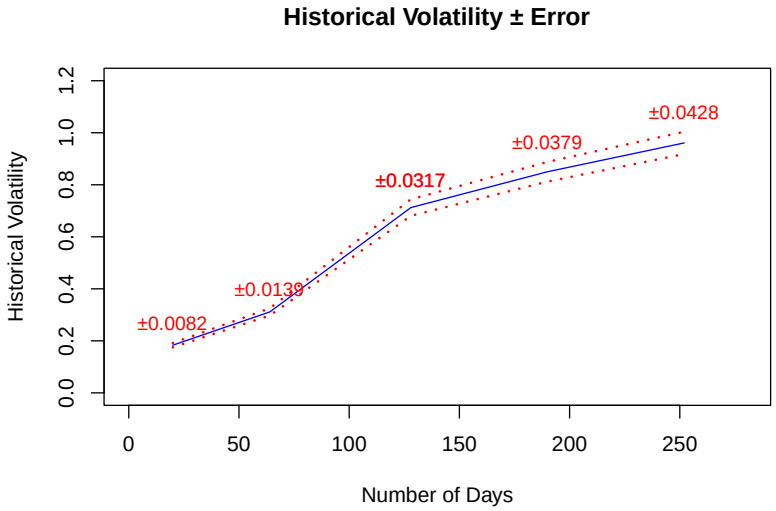
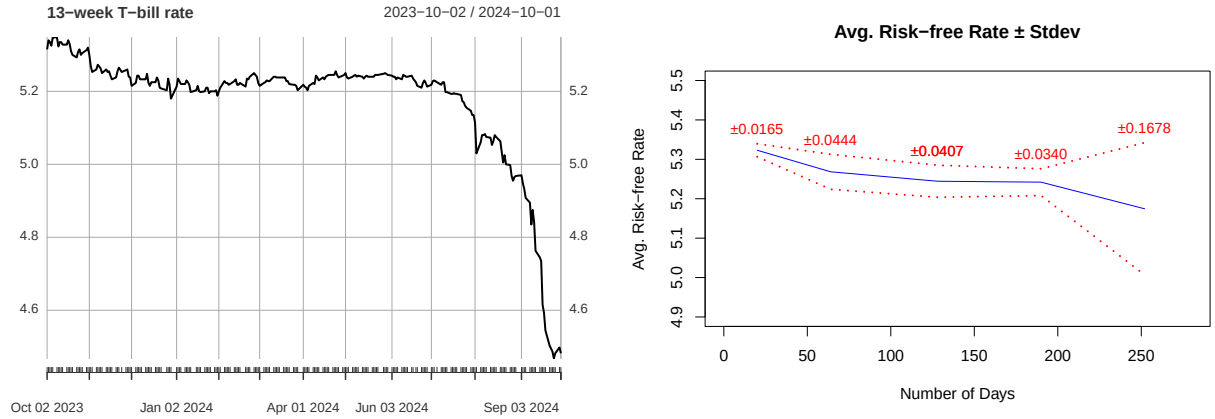


Figure 3: Hist Vol. over 3M intervals

Estimate risk-free rate r

The next parameter we estimate is the risk-free-rate, which we source from 13 week T-bill rates in ^IRX. In Fig 4. we observe that the risk-free rate $r = 4.482\%$ is a sharp decline near the end of our pricing period 01-Oct-2024. Rather than using the final risk-free rate, we look at the standard deviations for risk-free rate along with a cumulative mean, shown below. We will use this adjusted risk-free-rate in order to model our options on the whole time period rather than the skew in risk-free rate at 01-Oct-2024.



(a) IRX Rates

(b) Avg. Risk-free rate Lookback

Figure 4: 13-week T-bill rate from IRX

From Fig 4.b, we observe that the optimal risk-free rate that describes option pricing over the entire period is at $r = (5.242 \pm 0.034)\%$. This captures a majority of the period that we are pricing options on as well as maintaining a low standard deviation across the rest of the rest of the risk-free rates.

$$r = 5.242\%$$

We will use this best average risk-free rate across to price American options.

3 American options on SMCI

SMCI options are traded on several days of the year. As of 02-Oct-2023 the first three maturities periods are:

Table 1: Expiry Dates and Corresponding Maturities

Expiry Date	Maturity Period	T
16-Oct-2024	14	0.056
05-Nov-2024	34	0.135
25-Nov-2024	54	0.214

These are the next business days when the options for SMCI mature.

3.1 Stability of Put-Call Parity

Next we observe the stability of put-call parity inequality for American options. Recall the put-call parity for American Options with no Dividends from Section 1. Since the expiration dates as seen in Table 1. have a short duration, we can reduce the time constant for small values $e^{-rT} \approx 1$, this reduces the American option inequality to:

American Put-Call Parity, Short Maturity Approx. ($D = 0$): $C - P \approx S_0 - K$

We test this approximation by calculating the difference between call and put, along with options pricing at-the-money $K = S_0$. This approximation proves to holds well as seen in Fig.5.

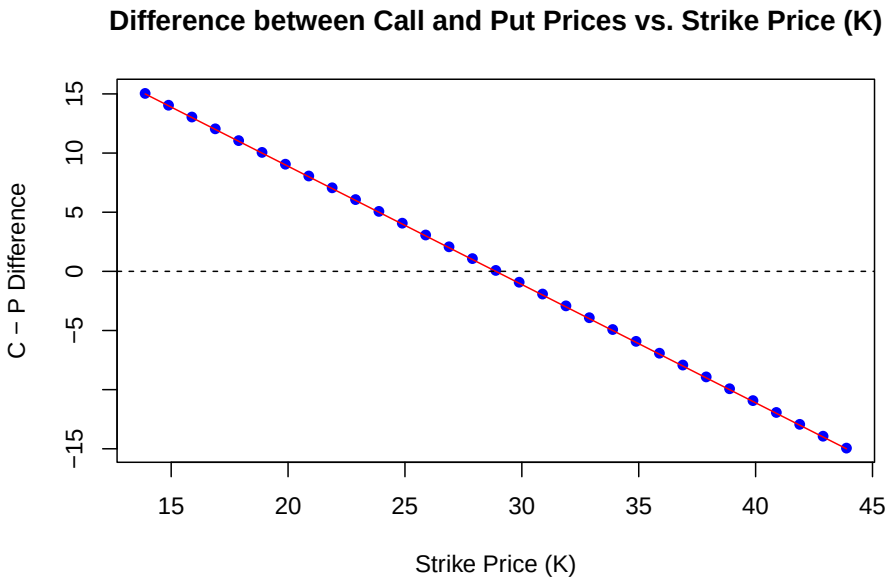


Figure 5: American Call - Put (blue) vs $S_0 - K$ (red)

The next step in validating binomial tree pricing for our options is to run a convergence test, this allows us to see how many iterations are required to find a good estimate to the Black-Scholes model. We observe that for a low $n = 4$, gives us a much more varied result as compared to using higher time steps $n = 100$.

For put prices at $n = 4$ we get:

$$P(K = S_0; n = 4) = 2.4201071$$

However, increasing the number of time steps, we expect a better approximation of the Black-Scholes model, for $n = 100$, the put price we estimate is:

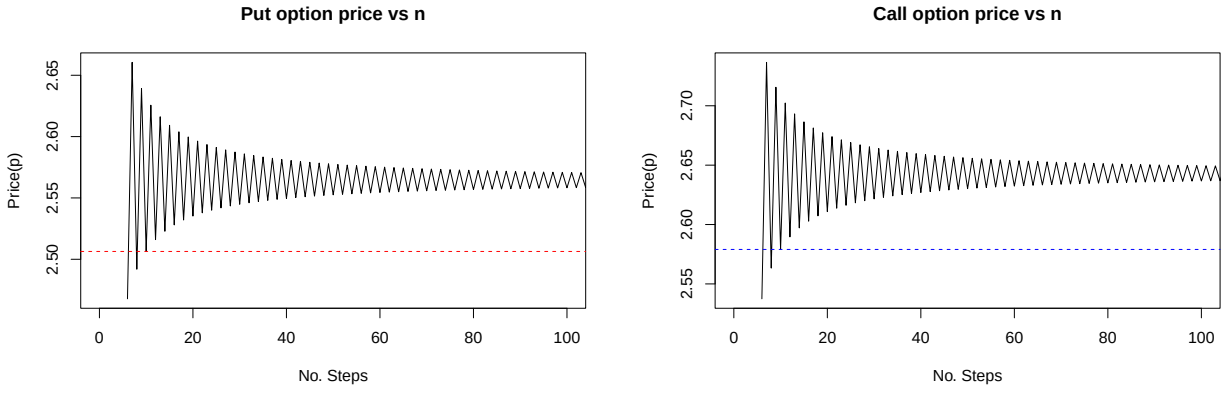
$$P(K = S_0; n = 100) = 2.5582479$$

3.2 Convergence Study

Now, Lets observe the effects of convergence over $n = 1 \cdots 100$, this will provide the effects of using higher binomial tree pricing time steps to approximate the Black-Scholes model. As expected, Fig 6. shows that as the Call/Put price time steps increasing, we converge to a central point. This point is a strong estimate of Black-Scholes and with higher n

$$BT_{C/P}(n \to \infty) = BS_{C/P}$$

As we can see, at around $n > 80$, the Binomial Tree pricing method reaches a strong approximation for Black-Scholes, and we are able to compute options pricing well using this estimate value. For American options pricing we will use $n = 100$ for our approximations.



(a) Puts

(b) Calls

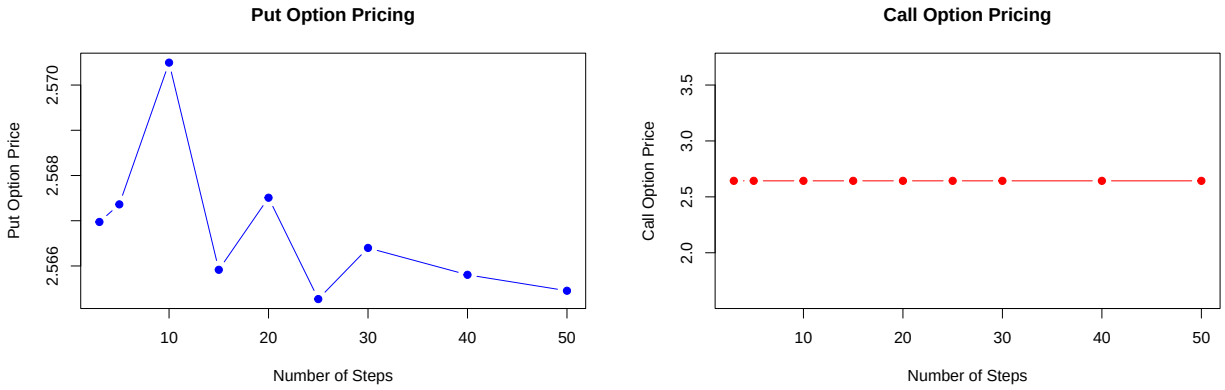
Figure 6: Put-Call Convergence at $n = 1 \cdots 100$, Dotted line is initial estimate at $n = 10$

3.3 Control Variate Method for Reducing Computation Complexity

One last approach we will use to price options is the Control Variate method. For this approximation we will get the price of options via Europeans f_E, Americans f_A, along with Black-Scholes European price for options f_{BS}. This arrives us at an approximation for pricing:

$$f_{CV} = f_A + (f_{BS} - f_E)$$

The goal of using Control Variate is to find a strong approximation of Black-Scholes without requiring a large n, we sample different values of n at the money $K = S_0$, to get a fast approximation that fits with Black-Scholes American options.



(a) Puts (b) Calls

Figure 7: Control Variate Convergence increasing sizes n

As seen above, Control Variate allows us to get a highly accurate estimate for the options pricing in fewer iterations. Instead of calculating $n = 80$ steps of Binomial Tree estimation for American Options, we can get away with $n = 20$ for both American and European calculation to 3 decimals of accuracy, plus a single calculation of European Black Scholes. This reduces the computation complexity from $n = 80 \rightarrow 41$ calculations, while getting better results overall. However, as seen in Fig 7b. we can further reduce the operation

count of Control Variate, the convergence of Call in n steps happens almost instantly. This matches how call options are treated in American options with no dividends. As mentioned previously, reducing the e^{-rT} term for short-term maturities gives us:

$$\text{American Put-Call Parity, Short Maturity Approx. } (D = 0)\text{: } C - P \approx S_0 - K$$

This is the same put-call parity for European options.

$$\text{European Put-Call Parity, Short Maturity Approx. } (D = 0)\text{: } c - p \approx S_0 - K$$

Accordingly, we reduce the iteration count for call options to $n = 5$, for a total of $n = 20 + 5 + 1 = 26$ computations, resulting in a $3.077\times$ reduction in time steps to calculate an accurate estimate for call and put options. We will use Control Variate Method with Puts $n = 20$ and Calls $n = 5$ for calculating Greeks.

4 Greeks

4.1 Finite Difference Method

Now we will look at our method for calculating options and apply it to Greeks calculations. For this approach we use the Finite Differences method, namely we look at how the American call-put options change over K through Delta and Gamma.

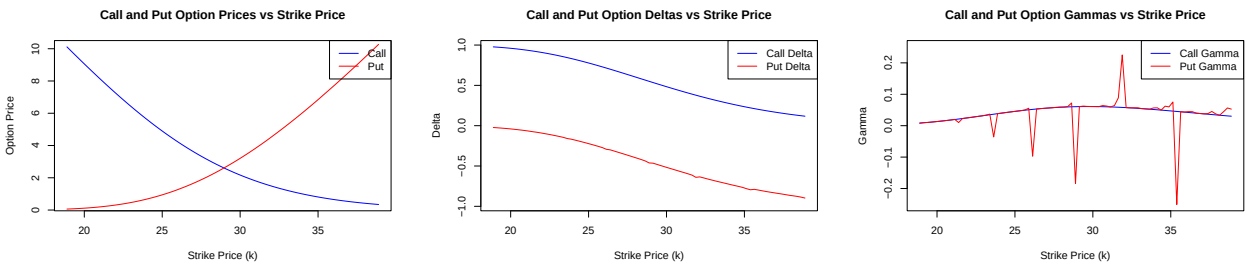
The Finite Difference method for Delta, given a step size of $S_0 \to S_0 + \delta$:

$$\Delta = \frac{P(S_0 + \delta) - P(S_0 - \delta)}{2\delta}$$

And Gamma with step size $S_0 \to S_0 + \delta$

$$\Gamma = \frac{P(S_0 + \delta) - 2P(S_0) + P(S_0 - 1)}{\delta^2}$$

For the calculations, we will use the earliest maturity date at 16-Oct-2024 of $T = 14/252$ years. We will calculate with step size $\delta = 0.1$, and sample every 0.25 steps centered at-the-money $K = S_0 = 28.886$. The resulting Greeks calculation shows us expected results. As seen in Fig 8. as Call options decreases, Put increases, hitting each other at the strike price $K = S_0$ at-the-money. Delta calculations are smoothly decreasing, Call in range [0, 1], while Puts [-1, 0]. However, we see the effects of numerical instability in finite differences with the Gammas calculation. Since this functions approximates a 2nd derivative, the result is volatile Gammas along the Put prices. As mentioned previously, we set Puts in the Control Variate method to $n = 20$, which was a strong approximation as in Fig 7. But in some cases of pricing, the approximation is not enough and still results in large spikes in data.



(a) Option Price (b) Delta (c) Gamma

Figure 8: Greeks Calculated via Control Variate and Finite Difference

4.2 Reducing Tweak size δ

To handle the spikes in data, our first approach is to change the δ size of Gamma. The effects of reducing δ is shown in Fig 9. The smaller of δ, improves numerical stability, but we still have an issue with Gamma at-the-money $K = S_0$, this value gets much worse the higher our δ.

We see that from the three values of δ, the most stable values are in $\delta = 0.01$, this balances

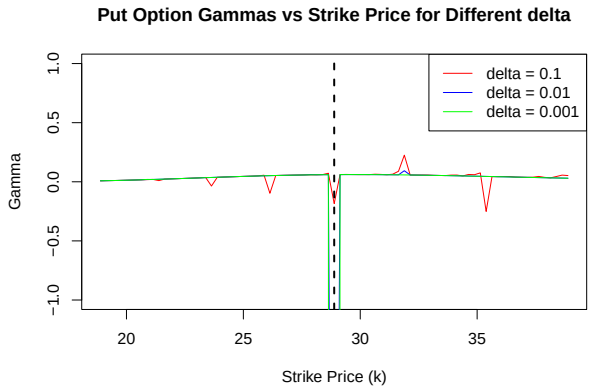


Figure 9: Put Gammas with Different Step Sizes, S_0 as Y-line

the numerical stability error found with $\delta = 0.1$ and the large error at-the-money from $\delta = 0.001$.

4.3 Regression Smoothing for Gamma

To improve our Gamma calculation, we need a smoothing method that both fixes the numerical instability at-the-money while representing the highest point in Gamma. The next approach we will use is polynomial smoothing, which is using regression to approximate values from a sample dataset. In the case of $k = 1$ this is linear regression, however increasing the polynomial size $k = 2$, we can find a function that matches the Gammas prices closely. However, to benefit from this approximation, we remove several points near at-the-money $K = S_0$, this ensures the regression algorithm is not skewed to large numerical errors. As seen in Fig.10, we have significantly better results for Gamma, that are non-zero and peak at-the-money.

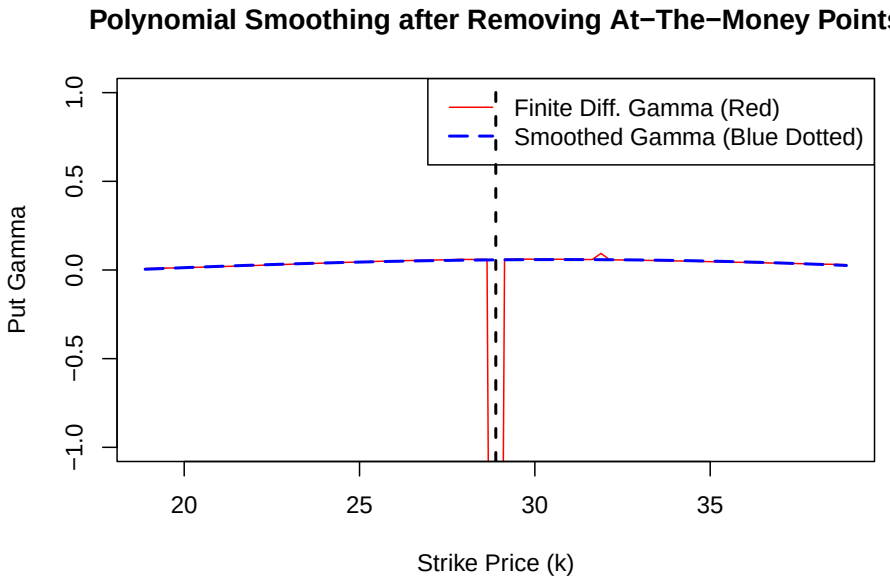


Figure 10: Polynomial k=2 Smoothing Put Gammas, S_0 as Y-line

Our final calculations for Greeks will be Finite Differences method along with $k = 2$ regression smoothing for Gammas using $\delta = 0.01$.

5 Hedging

The closing prices of SMCI until the first expiration date at 16-Oct-2024 is shown in Fig 11.

```
##            SMCI.Close
## 2023-10-02     28.886
## 2023-10-03     26.997
## 2023-10-04     28.040
## 2023-10-05     28.760
## 2023-10-06     28.967
## 2023-10-09     29.000
## 2023-10-10     31.031
## 2023-10-11     30.886
## 2023-10-12     29.321
## 2023-10-13     28.520
## 2023-10-16     29.169
```

Figure 11: SMCI Closing Dates

For the hedging exercise, we consider a put option with strike $K = 31$, we test this with the last 4 days until the first expiration date 16-Oct-2024. For each day, we compute the option price and its Delta using the binomial tree model. From the Deltas we create a dynamically hedged portfolio, where we purchase Δ shares of SMCI. The daily price change of the hedged portfolio then becomes:

$$\Pi(t) - \Pi(t-1) = P(t) - P(t-1) - \Delta_P(t-1)(S(t) - S(t-1))$$

Note that since we are using the binomial tree method to hedge put and deltas, there is a risk in hedging due to the numerical stability issue as mentioned in the previous section.

However, the results display the change in puts as the hedge reaches it's maturity, Delta approximately hits -1.0 as expected. And we are able to create the hedging strategy for put options in SMCI, given our strike price $K = 31$.

Table 2: Hedging Exercise with Strike $K = 31$

Day	Maturity	S_0	Put Price	Delta	$P(t) - P(t-1)$	$\Pi(t) - \Pi(t-1)$
2023-10-10	4 / 252	31.031	0.5376	-0.4258	NA	NA
2023-10-11	3 / 252	30.886	0.5929	-0.4638	0.0553	0.1413
2023-10-12	2 / 252	29.321	1.6790	-0.9230	1.0861	3.7137
2023-10-13	1 / 252	28.520	2.4800	-1.0000	0.8010	2.7875
2023-10-16	NA	29.169	1.8310	-0.9586	-0.6490	-1.8373

6 Next Steps

Handling options pricing in a volatile market opens alot of opportunities to how to manage volatility. As the paper suggestions, approaches such as regression to improve numerical stability become necessary to price Greeks in a volatile market. However if we had more time the following are some directions we could pursue:

- Effects of the rest of the option Greeks, Theta, Vega, Rho.
- Building a portfolio under high volatility market
- Other approaches for calculating Greeks with higher accuracy

7 Appendix

Work Completed

- Christopher Tarabay: Market Data Analysis, Write-up
- Hitanshu Reddy: Analysis of Put-Call Parity
- Vineel Pentrala: Analysis of Hedging

References

[1] Nicolaivicol. (n.d.). GitHub - nicolaivicol/binomial-tree-options-R: Pricing options via binomial trees: European, American, Chooser, Knock-Out, Average Strike. GitHub. https://github.com/nicolaivicol/binomial-tree-options-R

[2] J. Hull, Options, Futures and Other Derivatives, Pearson 10th Edition